Int. J. Heat Mass Transfer. Vol. 24, pp. 69–79
Pergamon Press Ltd. 1981. Printed in Great Britain

0017-9310/81/0101-0069 $02.00/0

FILM BOILING ON A HORIZONTAL PLATE — NEW CORRELATION

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(*Received* 25 *February* 1980)

Abstract — An approach to pool film boiling on a horizontal surface on the basis of the Reynolds analogy is suggested. Within the framework of the model considered, the influence of vapour film thickness and vapour velocity on Taylor instability of the interface has been investigated. Four limiting solutions have been obtained for laminar and turbulent vapour flow in the film with and without allowance for friction at the liquid–vapour interface. The relations suggested practically correlate all of the available experimental data with an accuracy of ±25%. The boundary of the region where heat-transfer rate depends on the size of the heating surface has been established and an empirical formula allowing for this effect has been obtained.

NOMENCLATURE

b, wave growth parameter;
c, wave velocity;
c_f, friction coefficient;
c_p, specific heat at constant pressure;
D, minimum size of heating surface;
g, acceleration of body forces;
Ga, $= \dfrac{g l_{cr}^3}{2}$, Galileo number;
i, imaginary unit;
l, instability wavelength;
l_{cr}, critical wavelength of two-dimensional instability;
l_D, $= 2\pi \sqrt{\left(\dfrac{3\sigma}{g(\rho' - \rho)}\right)}$, 'most dangerous' wavelength of two-dimensional instability;
l_{D_∞}, $= 2\pi \sqrt{\left(\dfrac{6\sigma}{g(\rho' - \rho)}\right)}$, 'most dangerous' wavelength of three-dimensional instability;
m, wave number;
Nu, $= \dfrac{\alpha l_{cr}}{\lambda}$, Nusselt number;
P, pressure;
Pr, Prandtl number;
q, heat flux density;
r, latent heat of vaporization;
r_*, effective heat of vaporization;
R, radius;
Ra, $= \dfrac{g l_{cr}^3}{\nu^2} Pr(\rho'/\rho - 1)$, modified Rayleigh number;
St, $= \dfrac{\alpha}{\rho c_p w}$, Stanton number;
T, $= T_w - T'$, temperature difference;
w, velocity.

Greek symbols

α, heat-transfer coefficient;
δ, film thickness;
σ, surface tension;
λ, thermal conductivity;
μ, dynamic viscosity;
ν, kinematic viscosity;
ρ, density;
τ, time.

Superscript

$'$, refers to liquid.

Subscripts

cr, critical;
D, 'most dangerous';
w, wall;
cr_2, film boiling crisis.

INTRODUCTION

FILM boiling on a horizontal plate has been the object of investigation since the end of the fifties. The first step in this direction seems to have been made by Chang Yan-Po [1] who, on the assumption of the analogy between free convection and film boiling, has derived the relationship of the type

$$Nu = 0.295\left(Ra \frac{r_*}{c_p \Delta T}\right)^{1/3}. \tag{1}$$

A few years later, Berenson [2] applied the Taylor hydrodynamic instability theory to film boiling analysis. His work is based on the assumption of a laminar flow in the vapour film and of regular distribution of nucleation sites. Assuming the distance between the bubbles and their departure diameters to be proportional to the critical wavelength of the Taylor instability, the author obtained the equation

$$Nu = 0.672\left(Ra \frac{r_*}{c_p \Delta T}\right)^{1/4}. \tag{2}$$

Ruckenstein [3] has made an attempt to develop Berenson's theory. However, his final expression differs from (2) only by a constant coefficient, the value of which the author failed to determine.

Hamill and Baumeister [4] also considered the regular structure of distribution of nucleation sites, but, unlike the authors of [2] and [3], they determined the spacing between the bubbles and their departure diameters not *a priori* but on the basis of the principle of the system maximum entropy growth. As a result they obtained

$$Nu = 0.648\left(Ra\frac{r_*}{c_p\Delta T}\right)^{1/4}. \qquad (3)$$

Here the effective heat of vaporization $r_* = r + 0.95\, c_p\Delta T$ in contrast to all the other papers cited, where it was calculated simply from the arithmetic mean value of the vapour superheat, i.e. $r_* = r + 0.5\, c_p\Delta T$.

Papers [2–4] assumed laminar flow in the vapour film. In contrast to this, several equations have been proposed for turbulent film boiling. As is shown by the authors of [5], a turbulent film boiling model with an irregular chaotic distribution of nucleation sites must lead to the expression

$$Nu = 0.20\left(Ra\frac{r_*}{c_p\Delta T}\right)^{1/3} \qquad (4)$$

(with the constant fitted to experimental data).

Besides equation (4), there are also empirical correlations for the turbulent film boiling suggested by Clark [6] and Lao *et al.* [7]

$$Nu = 0.012\left(Ra\frac{r_*}{c_p\Delta T}\right)^{1/2}, \qquad (5)$$

$$Nu = 185\, Pr\left(\frac{r}{c_p\Delta T}\right)^{-0.09}. \qquad (6)\dagger$$

It should be noted that none of the relations given above is universal enough; all of them are mainly based on the experimental data of the authors themselves. Berenson's equation (2) is probably the most widely used and is recommended as the calculation equation in a number of recently published surveys and heat transfer handbooks [8, 9, 10]. However, the application of this equation leaves unsolved the following problems:

(1) Berenson's equation takes into account the existence of only a laminar flow in the vapour film. It is obvious that under certain conditions the laminar flow is replaced by a turbulent one. [Clark suggested this change to take place at $Ra(r_*/c_p\Delta T) > 10^7$.]

(2) To what extent the results of the Taylor theory [11] obtained for an isothermal flow can be applied to film boiling characterized by high temperature differences? Within his model, Berenson has got the answer of that being possible only in the region close to the film boiling crisis. However, formula (2) is recommended as a calculation equation for the entire range of film boiling temperatures. Hence, some discrepancy arises.

(3) Some publications [12, 13, 14] are available in which it is postulated that a decrease in the surface dimensions in a certain range leads to enhancement of heat transfer. The relationships available do not show how and from which magnitude the size of the heating surface starts to affect the rate of heat transfer. The only exception is paper [14] which correlates the results but only of a limited number of experiments.

In the present paper an attempt is made to derive a new relationship which would allow for all of the factors mentioned above.

FILM BOILING ON A HORIZONTAL PLATE. A BOUNDARY LAYER APPROACH

Consider a physical model of film boiling shown in Fig. 1. Let us assume that the sites of generation of bubbles are uniformly distributed above the heating surface in the corners of square meshes the sizes of which are determined by the Taylor instability. Vapour flows into the bubble from the area having an equivalent radius defined by

$$\pi R_2^2 = \tfrac{1}{2}\, l_D^2 = \tfrac{3}{2}\, l_{cr}^2. \qquad (7)$$

We shall consider a two-layer vapour film model consisting of a boundary layer and an external flow. The film thickness between the bubbles is constant; heat transfer from the bubble surface is neglected. Heat transfer from the wall to the liquid can take place either due to evaporation of liquid drops suspended in the vapour film or due to direct evaporation at the vapour–liquid interface. In the second case the exter-

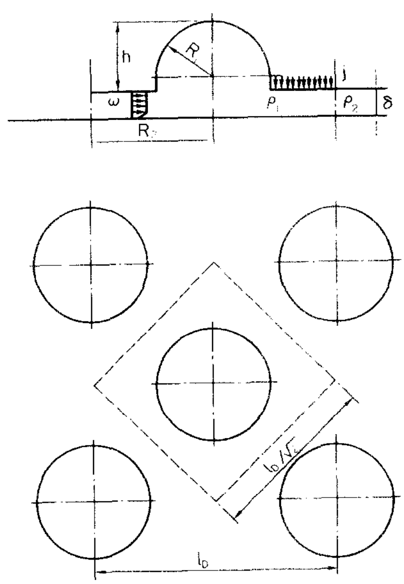

FIG. 1. A physical model of film boiling on a horizontal surface.

†In [7], the authors obtained the relationship $St' = 75.9\, Re^{-0.915}$, where $St' = (r/c_p\Delta T)$ and $Re = (l_D\, q_w/4\mu r)$, whence equation (6) can be readily obtained.

nal boundary of the vapour film can be considered as a permeable surface through which vapour is injected at the rate of $j \cong (q_w/r_*)$. Four different versions of solution of the heat transfer problem are possible depending on the vapour flow mode in the film and the injection rate at the external boundary.

Case 1. The flow in the film is laminar; the injection at the external boundary is neglected. In order to determine the heat transfer rate, we shall use the Reynolds analogy with Colburn's correction

$$St\,Pr^{2/3} = \frac{Nu}{Re\,Pr^{1/3}} = \frac{c_f}{2}, \quad (8)$$

where for the laminar flow

$$\frac{c_f}{2} = \frac{0.664}{Re^{1/2}}. \quad (9)$$

As a characteristic linear dimension the quantity $L = R_2 - R_1$ may be chosen while the velocity scale can be found from the equation of motion for the external flow

$$\frac{\rho w^2}{2} = \Delta P = P_2 - P_1. \quad (10)$$

The pressure drop ΔP is determined by the difference of hydrostatic pressures with regard for the pressure rise under a curvilinear surface

$$P_2 - P_1 = g(\rho' - \rho)h - \frac{2\sigma}{R_1}. \quad (11)$$

According to [2, 15, 16], the height and the radius of a bubble are proportional to the value of the critical wave length of the Taylor instability, i.e.

$$h = C_h l_{cr}, \qquad R_1 = C_R l_{cr}. \quad (12)$$

Combining (10), (11) and (12), we shall obtain

$$P_2 - P_1 = C_{\Delta P} g(\rho' - \rho) l_{cr}. \quad (13)$$

Substituting (13) into (10) yields the value of the velocity scale in the external flow

$$w = C_w \sqrt{\left[g\left(\frac{\rho'}{\rho} - 1\right) l_{cr}\right]}. \quad (14)$$

Equations (8) and (9), with account for (14) and

$$L = R_2 - R_1 = C_L l_{cr}, \quad (15)$$

will give the dimensionless heat-transfer coefficient

$$Nu = \frac{\alpha l_{cr}}{\lambda} = (\text{const})_1 \left[\frac{g l_{cr}^3}{\nu^2}\left(\frac{\rho'}{\rho} - 1\right)\right]^{1/4} \cdot Pr^{1/3}$$

or

$$Nu = (\text{const})_1 \left[Ga\left(\frac{\rho'}{\rho} - 1\right)\right]^{1/4} \cdot Pr^{1/3}. \quad (16)$$

Case 2. The flow in the film is turbulent; the injection at the external boundary is neglected.

For the turbulent mode of flow on a rough plate the friction coefficient can be taken constant if the condition that $L/R_\mu < 10^3$ is satisfied, where L is the plate length, R_μ is the mean value of roughness. For commercial surfaces $R_\mu = (5\text{–}10) \times 10^{-6}$ m, the plate 'length' in our case is $L = R_2 - R_1 \approx l_{cr} \approx 10 \times 10^{-3}$ m, thus $1/R_\mu \simeq 10^3$. Hence, equation (8) can be written as follows

$$St \cdot Pr^{2/3} = \frac{Nu}{Re \cdot Pr^{1/3}} \approx \text{const}. \quad (17)$$

Substituting the velocity and characteristic dimension from (14) and (15), we obtain

$$Nu = \frac{\alpha l_{cr}}{\lambda} = (\text{const})_2 \left[\frac{g l_{cr}^3}{\nu^2}\left(\frac{\rho'}{\rho} - 1\right)\right]^{1/2} \cdot Pr^{1/3},$$

or

$$Nu = (\text{const})_2 \left[Ga\left(\frac{\rho'}{\rho} - 1\right)\right]^{1/2} \cdot Pr^{1/3}. \quad (18)$$

Case 3. The flow in the film is laminar; vapour is injected at the external boundary at the rate of

$$j = \frac{q_w}{r_*}.$$

The equation of motion for the external layer, where viscosity forces are negligibly small, can be expressed as follows

$$\rho w = \frac{dw}{dx} = -\frac{dP}{dx} + \frac{d\tau}{dy}. \quad (19)$$

We shall consider the inertia term to be small and the shear stresses to be conditioned by the presence of injection, i.e.

$$\tau = jw. \quad (20)$$

Then

$$\Delta P \sim \int_{R_1}^{R_2} \frac{d(jw)}{dy} dx = \frac{d}{dy}(j\langle w\rangle L) \sim j\langle w\rangle \frac{L}{\delta}.$$

Let us suppose that L/δ is a constant. Then the velocity scale can be found from

$$\langle w\rangle \sim \frac{\Delta P}{j} = \frac{\Delta P r_*}{q_w} = \frac{\Delta P r_*}{\alpha \Delta T}. \quad (21)$$

Combining equations (8), (9), (15) and (21) and accomplishing several transformations, we shall arive at

$$Nu = (\text{const})_3 \left[\frac{g l_{cr}^3}{\nu^2}\left(\frac{\rho'}{\rho} - 1\right)\right]^{1/3} \times \left(\frac{r_*}{c_p \Delta T}\right)^{1/3} \cdot Pr^{5/9}, \quad (22)$$

or

$$Nu = (\text{const})_3 \left[Ga\left(\frac{\rho'}{\rho} - 1\right)\right]^{1/3} \times \left(\frac{r_*}{c_p \Delta T}\right)^{1/3} \cdot Pr^{5/9}. \quad (23)$$

Case 4. The flow in the film is turbulent; vapour is uniformly injected at the rate of $j = (q_w/r_*)$ at the external boundary.

Using the Reynolds analogy in the form of (17) and the expressions for the velocity scale (21) and linear dimension (15) and accomplishing a number of transformations, we obtain

$$Nu = (\text{const})_4\left[Ga\left(\frac{\rho'}{\rho} - 1\right)\right]^{1/2} \times \left(\frac{r_*}{c_p\Delta T}\right)^{1/2} \cdot Pr^{2/3}. \quad (24)$$

The values of constants in equations (16), (18), (23), (24) can be determined from the experiment. Further, since at present there is no sufficiently accurate method to calculate the 'effective' heat of vaporization, r_*, we shall consider the ratio $(r_*/c_p\Delta T)$ to be a certain function of the criterion $(r/c_p\Delta T)$, i.e.

$$\frac{r_*}{c_p\Delta T} = f\left(\frac{r}{c_p\Delta T}\right). \quad (25)$$

The particular form of the function f will be defined below from comparison with the experimental data.

THE CRITICAL AND MOST DANGEROUS WAVE LENGTH IN FILM BOILING ON A HORIZONTAL SURFACE

When deriving the heat transfer equations (16), (18), (23), (24), we considered the critical and 'most dangerous' wavelength of the Taylor instability to be constant values equal to $2\pi\sqrt{[\sigma/g(\rho'-\rho)]}$ and $2\pi\sqrt{3}\sqrt{[\sigma/g(\rho'-\rho)]}$, respectively. Physically this assumption implies that a vapour film is rather thick and the vapour velocity in it is equal to zero. Now we shall check the validity of these assumptions.

The general solution of the kinematic equation for the liquid–vapour cocurrent eddy-free flow with the liquid located above the vapour is expressed [17] as

$$m\rho(w-c)^2\coth(m\delta) + m\rho'(w'-c)^2\coth(m\delta') = \sigma m^2 - g(\rho'-\rho). \quad (26)$$

For a sufficiently thick layer of a stagnant liquid

$$w' = 0 \quad \text{and} \quad \coth m = 1.$$

Then

$$m\rho(w-c)^2\coth(m\delta) + m\rho' c^2 = \sigma m^2 - g(\rho'-\rho), \quad (27)$$

where

$$c = \frac{n}{m}, \qquad b = -in. \quad (28)$$

For a sufficiently thick vapour film and zero vapour velocity in it, equation (26) goes over into

$$m\rho c^2 + m\rho' c^2 = \sigma m^2 - g(\rho'-\rho), \quad (29)$$

whence, after accomplishing simple transformations and allowing for (28), we get the value of the growth parameter

$$b = \left[\frac{mg(\rho'-\rho)}{\rho'+\rho} - \frac{\sigma m^3}{\rho'+\rho}\right]^{1/2}. \quad (30)$$

The critical wavelength is the name given to that value of the wavelength above which the occurrence of instability is observed; mathematically, this instant is consistent with $b = 0$. From equation (30) the following equation can be easily obtained

$$l_{cr} = \frac{2\pi}{m_{cr}} = 2\pi\sqrt{\left[\frac{\sigma}{g(\rho'-\rho)}\right]}. \quad (31)$$

The 'most dangerous' wavelength is the one which grows at the maximum rate, i.e. satisfies the condition $(\partial b/\partial m) = 0$. Performing the differential operation on (30) and equating the result to zero will yield

$$m_D = \sqrt{\left[\frac{g(\rho'-\rho)}{3\sigma}\right]} \quad (32)$$

and

$$l_D = \frac{2\pi}{m_D} = 2\pi\sqrt{3}\sqrt{\left[\frac{\sigma}{g(\rho'-\rho)}\right]}. \quad (33)$$

It should be noted that here the maximum value of the growth parameter is found by substituting (32) into (30)

$$b_{max} = \frac{(12)^{1/4}}{3}\left\{\frac{[g(\rho'-\rho)]^{3/2}}{\sigma^{1/2}(\rho'+\rho)}\right\}^{1/2}. \quad (34)$$

Equations (32) and (33) determine the 'most dangerous' wave number and wavelength when the vapour film thickness and vapour velocity are left out of account. In order to allow for their effect, it is necessary to solve equations (27)–(28) for the growth parameter b.

When $m\delta < 0.4$, it can be presumed with sufficient accuracy that $\coth(m\delta) \approx (1/m\delta)$. Then equation (27) can be rewritten as

$$m\rho(w-c)^2\frac{1}{m\delta} + m\rho' c^2 = \sigma m^2 - g(\rho'-\rho). \quad (35)$$

Substituting the values of c and n from equation (28) and performing all the necessary transformations we shall arrive at a quadratic equation in b the solution of which is

$$b_{1,2} = -\frac{m\rho w}{\rho + m\rho'\delta}i \pm \sqrt{\left[\frac{g\delta(\rho'-\rho)m^2}{\rho+m\delta\rho'} - \frac{\delta m^3(m\delta)}{\rho+m\delta\rho'} + \frac{\rho'\rho m^2(m\delta)w^2}{(\rho+m\rho'\delta)^2}\right]}. \quad (36)$$

The imaginary part of equation (36) is a harmonic component, i.e. it represents steady oscillations. Of principal interest to us is the second term which, at certain values of the parameters entering it, can acquire real positive values, the latter corresponding to the onset of unsteady oscillations of the interface.

Let us rewrite the real part of equation (36) in the form

$$b = \left[\frac{g(\rho' - \rho)m}{\rho/m\delta + \rho'} - \frac{\sigma m^3}{\rho/m\delta + \rho'} + \frac{\rho'\rho w^2 m^2}{(\rho/m\delta + \rho')^2 m\delta}\right]^{1/2}. \quad (37)$$

For the convenience of further calculations, we introduce the following dimensionless variables $B = (b/b_{max})$ and $N = m/m_D$, where b_{max} and m_d are determined by equations (34) and (32), respectively.

In addition, by virtue of equations (10) and (13)

$$\rho w^2 \sim g(\rho' - \rho)l$$

or

$$\rho w^2 = (\text{const})\frac{g(\rho' - \rho)}{m}, \quad (38)$$

where the constant is of the order of unity.

Then equation (37) can be represented as

$$B = \left[\frac{3}{2}M\frac{\rho'/\rho + 1}{\rho'/\rho + \dfrac{1}{M\delta m_D}} - \frac{1}{2}M^3\frac{\rho'/\rho + 1}{\dfrac{1}{M\delta m_D}} + \frac{3}{2}(\text{const})\frac{\dfrac{\rho'}{\rho}\left(\dfrac{\rho'}{\rho} + 1\right)}{\left(\dfrac{\rho'}{\rho} + \dfrac{1}{M\delta m_D}\right)^2}\frac{1}{\delta m_D}\right]^{1/2}. \quad (39)$$

It should also be noted that with disregard of the vapour film thickness and the vapour velocity in it, equation (39) goes over into the equation

$$B = \left[\frac{3}{2}M\left(1 + \frac{\rho}{\rho'}\right) - \frac{1}{2}M^3\left(1 + \frac{\rho}{\rho'}\right)\right]^{1/2}, \quad (40)$$

which has the maximum at point $B = 1$, $M = 1$.

In order to evaluate the influence of film thickness and vapour velocity on the 'most dangerous' wavelength it is necessary to clarify the manner in which the maximum of the curve $B = B(M)$ is displaced when the parameters ρ'/ρ and δm_D are changed. The lack of data on the vapour film thickness has made it necessary to resort to the estimation of the minimum value of this quantity by means of a simple relation

$$\delta_{min} \simeq \frac{\lambda}{\alpha_{cr_2}}. \quad (41)$$

Obviously, this evaluation will provide the thickness of a thermal boundary layer, while it may happen that the total thickness of the vapour film will be somewhat higher.

Table 1 presents the values of δ_{min} and $(\delta m_D)_{min}$ calculated on the basis of experimental data of a number of papers. We can conclude that in actual systems one should expect the values of δm_D to be of the order of 0.03 and above.

In Fig. 2, the dimensionless growth parameter is shown as a function of the dimensionless wave number within the range of the parameters ρ'/ρ from 10 to 1000 and of δ_{m_D} from 0.03 to ∞; the range practically covers the entire presumable region of film boiling at pressures not very close to the critical one [the data are given which were calculated with the constant in equation (39) equal to 0.5]. It is obvious that a decrease in the liquid to vapour density ratio as well as in the vapour film thickness lead to the displacement of the maximum of the curves $B(M)$ into the region of higher wave numbers. As this takes place, the 'most dangerous' wavelength decreases correspondingly It must also be emphasized that a decrease in the film thickness and an increase in ρ'/ρ lead to blurring of the maximum into rather a broad range of the wave numbers (or wavelengths). Physically, this result means that in film boiling, unlike the isothermal process of instability, there must be a whole set of 'most dangerous' wavelengths, determining different values of spacings between the bubbles, their departure diameters, etc., which is convincingly confirmed by the experiment [16].

Thus, strictly speaking, in the functional relation of the type

$$Nu = (\text{const})\left[\frac{gl_{cr}^3}{\nu^2}\left(\frac{\rho'}{\rho} - 1\right)\right]^n Pr^{1/3},$$

the critical wavelength associated, via the constant coefficient, with the 'most dangerous' wavelength

Table 1. Minimum values of vapour film thickness

Liquid	$\delta_{min} \times 10^3$ (m)	$m_D \times 10^{-3}$ (m^{-1})	$(\delta m_D)_{min}$	Ref.
Hydrogen	0.223	0.34	0.076	[18]
Helium	0.039	1.98	0.077	[19]
Freon-113	0.0494	0.56	0.0276	[12]
Freon-113	0.0756	0.56	0.0424	[20]
Pentane	0.0833	0.373	0.0311	[21]
Freon-II	0.054	0.514	0.0277	[16]
Water	0.143	0.23	0.0329	[16]

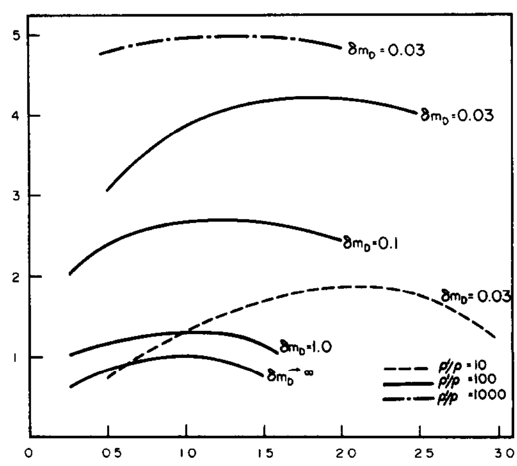

FIG. 2. Growth parameter vs wave number for different film thicknesses with vapour velocity taken into account.

appears to be the function of the film thickness δ which, in turn, is obviously a function of the temperature difference ΔT and the density ratio ρ'/ρ. Figure 2 shows that for very thick films $M_D \to 1$, i.e. the 'most dangerous' wavelength

$$l_D \to \frac{2\pi}{M_D m_D} = 2\pi\sqrt{3}\sqrt{\left[\frac{\delta}{g(\rho'-\rho)}\right]}$$

(when a three-dimensional instability is considered, then, according to [7], the factor $\sqrt{3}$ is replaced by $\sqrt{6}$). On the other hand, when $\delta m_D \to 0$, equation (39) goes over into

$$B = \left[\frac{3}{2}M^2\left(\frac{\rho'}{\rho}+1\right)\delta m_D - \frac{1}{2}M^4\left(\frac{\rho'}{\rho}+1\right)\delta m_D + \frac{3}{2}(\text{const})\frac{\rho'}{\rho}\left(\frac{\rho'}{\rho}+1\right)M^2\delta m_D\right]^{1/2}, \quad (42)$$

whence, by equating dB/dM to zero, we can obtain

$$M_D = \frac{\sqrt{6}}{2}\left(1 + \text{const}\frac{\rho'}{\rho}\right)^{1/2} \quad (43)$$

and

$$l_D = \frac{2\pi\sqrt{3}}{\frac{\sqrt{6}}{2}\left(1+\text{const}\frac{\rho'}{\rho}\right)^{1/2}}\sqrt{\left[\frac{B}{g(\rho'+\rho)}\right]}. \quad (44)$$

Thus, the value of l_D determined with the film thickness and vapour velocity effects taken into account can, in principle, differ substantially from the values of l_D given by equation (33). This leads one to the question as to what extent such a discrepancy can influence the film boiling. Figure 3 presents a typical dependence of the 'most dangerous' wavelength on the vapour film thickness at $(\rho'/\rho) = 100$ (by the order of magnitude this corresponds to pentane or freon-113 boiling under atmospheric pressure). It is natural that, from the point of view of film boiling, we are interested only in that part of the curve which is located to the right of the line corresponding to the crisis of film boiling. Generally speaking, the 'most dangerous' wavelength should be described by the function of the type

$$\frac{l_D}{l_{D_\infty}} = \varphi\left[\delta(\Delta T), \frac{\rho'}{\rho}\right]. \quad (45)$$

The construction of such a relation poses some difficulty for the lack of sufficient information on the vapour film thickness and its change with an increase in the temperature difference. Fortunately, it appears that within the range of the vapour film thicknesses or temperature differences which correspond to the film boiling on a horizontal surface, the dependence of the 'most dangerous' wavelength on the vapour film thickness is rather weak (when δm_D is decreased by two orders of magnitude, l_D varies by only 40%). It should be noted that actually this dependence is still weaker, since, as has been stated above, equation (41) gives the least possible value of the film thickness. This provides good reasons for the use, to a first approximation, in relations of the type of (16), (18), (23), (24) of the expressions for the 'most dangerous' or critical wavelength which corresponds to the limiting case of $\delta m_D \to \infty$ and $w \to 0$, i.e. determined by equations (31), (33).

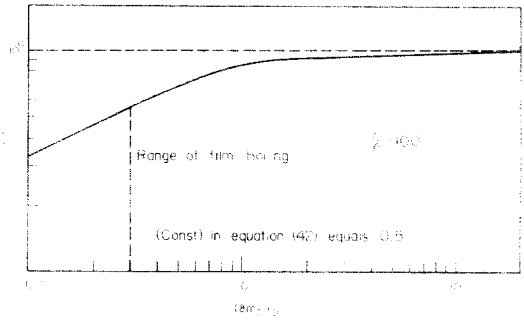

FIG. 3. Dependence of the 'most dangerous' wavelength on vapour film thickness.

HEAT TRANSFER CORRELATIONS

Before entering into comparisons between the experimental data and equations (16), (18), (23), (24), it is necessary to say a few words about the criteria for selecting the data to be compared:

(1) Equations (16), (18), (23), (24) have been obtained for 'large-sized' surfaces. As was shown earlier [14], a surface may be considered 'large' if its minimum size exceeds $2l_D$ †, i.e.

$$D \geq 4\pi\sqrt{6}\sqrt{\left[\frac{\sigma}{g(\rho'-\rho)}\right]}. \quad (46)$$

Most of the available experimental data have been obtained on 'large' surfaces, however, there are some well-known papers [12, 8, 22, 23] the data of which do not satisfy condition (46) and should be excluded from the analysis.

(2) It is known that in some cases impurities or low-conductivity coatings on the heating surface may lead to a marked improvement of heat transfer in the film region. This phenomenon is still not completely understood: the increase in the heat-transfer coefficient is credited to a contact heat exchange between the liquid and the wall, as well as to intensification of wave oscillations at the vapour–liquid interface. Neither of these effects is accounted for by the model suggested in the present paper, therefore the data obtained for the surfaces with coatings [20] and those obtained by the technique which could involve contamination of the heating surfaces [24] have been excluded from the analysis.

† Paper [14] considered a two-dimensional instability of vapour–liquid interface, which corresponds to the factor $\sqrt{3}$ instead of $\sqrt{6}$ in equation (46).

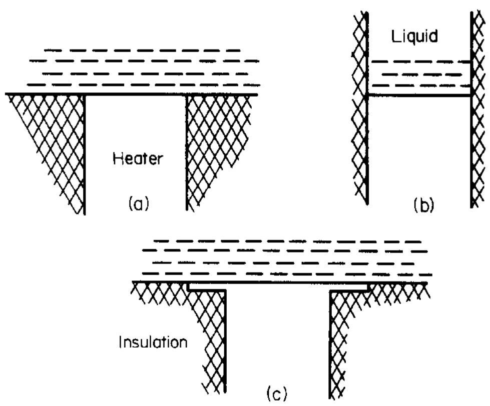

FIG. 4. Types of heating surfaces.

(3) The heat-transfer surfaces used in the experiments must satisfy the conditions T_w = const. or q_w = const. This requirement is met by the surfaces of the 'disk'- or 'plate'-type [Fig. 4(a)] and the 'bottom-of-vessel'-type [Fig. 4(b)] with steady-state (electrical, steam or gaseous) heating or unsteady cooling. Application of the surfaces of the 'disk with a diaphragm'-type [Fig. 4(c)] admits the existence of large radial temperature gradients and heat sinks which may distort the experimental results [5, 24].

(4) It is known that in generalized coordinates the data on film boiling of nitrogen are, as a rule, located much above those for other liquids. In our opinion, this problem is very important since, for a variety of reasons (first of all, safety and cheapness), it is the nitrogen which is the most used fluid for investigations of boiling. We think that a plausible reason for the anomalous behaviour of the data on nitrogen is a certain (frequently appreciable) amount of oxygen present, as a rule, in nitrogen which is used in experiments. Moreover, even an especially pure nitrogen, when used in an open Dewar vessel, becomes gradually enriched in oxygen from the air, with a binary mixture formed as a result. At the same time, it is known that binary mixtures exhibit appreciably higher film boiling heat transfer rates and that even a negligible amount (up to 1 %) of the second component can have a marked effect on heat transfer [25]. On account of this we had to eliminate from consideration those experimental data which had been obtained without special precautions to maintain the high purity of boiling nitrogen [14, 24].

Figure 5 presents in generalized coordinates the data on boiling of nine various liquids obtained by fifteen various research groups. Up to the value of $Ga[(\rho'/\rho) - 1] < 1 \cdot 10^8$, the experimental points are satisfactorily approximated by the expression of the type

$$Nu \simeq C_1\left[Ga\left(\frac{\rho'}{\rho} - 1\right)\right]^{1/3},$$

corresponding to the laminar flow in a vapour film; at $Ga[(\rho'/\rho) - 1] > 1 \cdot 10^8$, a turbulent regime appears to set in which is described by

$$Nu = C_2\left[Ga\left(\frac{\rho'}{\rho} - 1\right)\right]^{1/2}.$$

On the other hand, there is a considerable amount of data which deviate markedly from the approximating lines. This deviation can be attributed to the influence of friction on the vapour–liquid interface which can be taken into account by the criterion $r/c_p\Delta T$. Figures 6 and 7 show the same set of data but plotted separately for a laminar and a turbulent region as $Nu/\{Ga[(\rho'/\rho) - 1]\}^n$ vs $r/c_p\Delta T$. The experimental data can be approximated by relations (18), (23), (24)

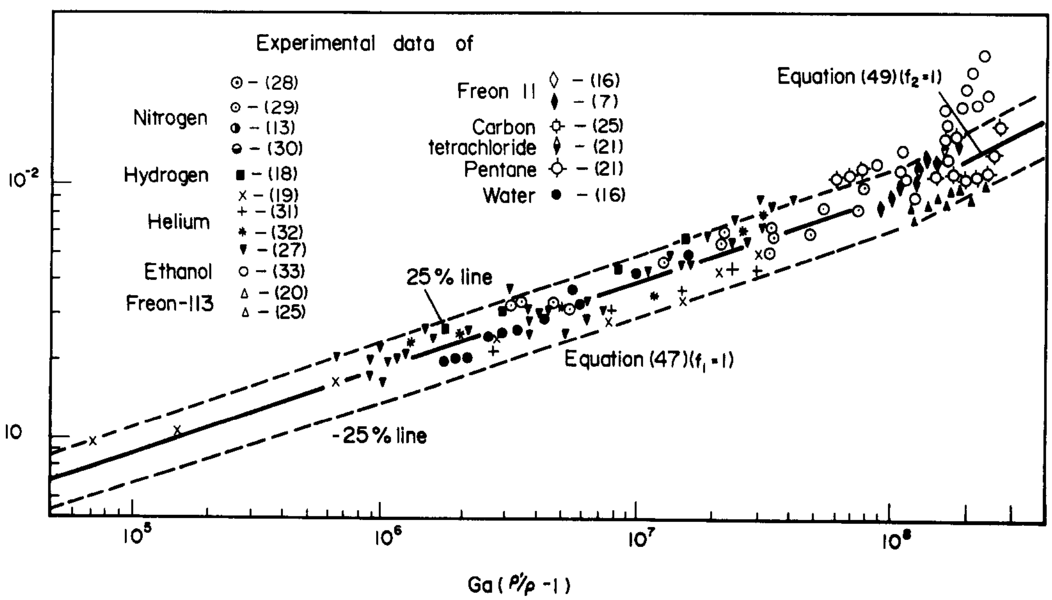

FIG. 5. Comparison of proposed relations with experimental data on film boiling on a horizontal surface.

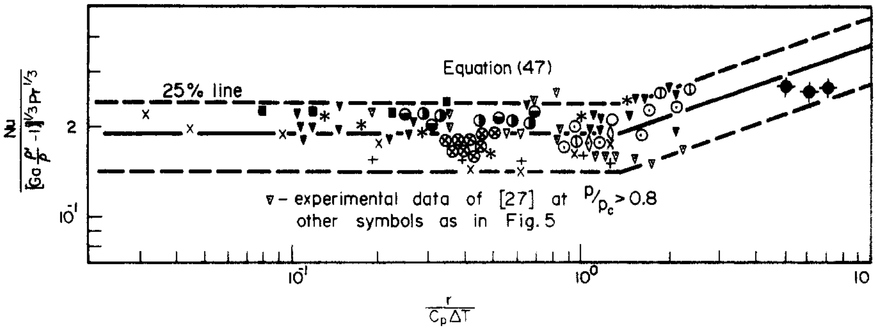

FIG. 6. Correlation of data on laminar film boiling.

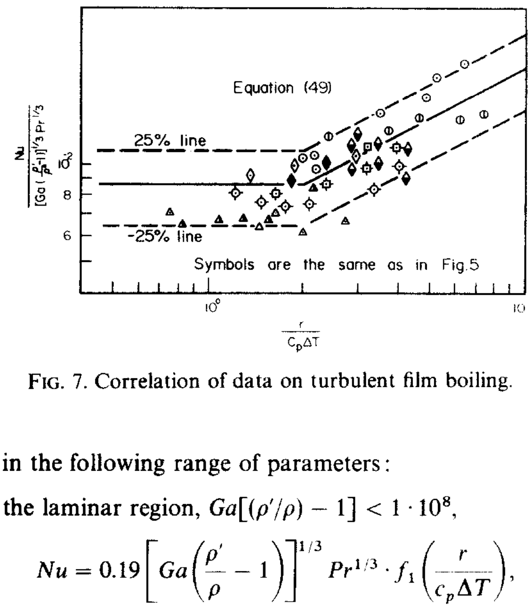

FIG. 7. Correlation of data on turbulent film boiling.

in the following range of parameters:

the laminar region, $Ga[(\rho'/\rho) - 1] < 1 \cdot 10^8$,

$$Nu = 0.19\left[Ga\left(\frac{\rho'}{\rho} - 1\right)\right]^{1/3} Pr^{1/3} \cdot f_1\left(\frac{r}{c_p \Delta T}\right), \quad (47)$$

where

$$f_1 = \begin{cases} 1 & \text{at } \dfrac{r}{c_p \Delta T} \le 1.4 \\ 0.89\left(\dfrac{r}{c_p \Delta T}\right)^{1/3} & \text{at } \dfrac{r}{c_p \Delta T} > 1.4 \end{cases}; \quad (48)$$

the turbulent region, $Ga[(\rho'/\rho) - 1] > 1 \cdot 10^8$,

$$Nu = 0.0086\left(Ga\frac{\rho' - \rho}{\rho}\right)^{1/2} Pr^{1/3} f_2\left(\frac{r}{c_p \Delta T}\right), \quad (49)$$

where

$$f_2 = \begin{cases} 1 & \text{at } \dfrac{r}{c_p \Delta T} < 2.0 \\ 0.71\left(\dfrac{r}{c_p \Delta T}\right)^{1/2} & \text{at } \dfrac{r}{c_p \Delta T} > 2.0 \end{cases} \quad (50)$$

(for the convenience of approximation the exponents on the Prandtl number have been chosen the same). Thermal properties of vapour refer to the average temperature $\bar{T} = T_s + \frac{1}{2}\Delta T$.

Equations (47) and (49) enable one to correlate, with an accuracy of $\pm 25\%$, almost all of the reported experimental data obtained within the following range of parameters:

$$Ga\left(\frac{\rho'}{\rho} - 1\right) = 7 \cdot 10^4\text{–}3 \cdot 10^8;$$

$$Pr = 0.69\text{–}3.45; \quad \frac{r}{c_p \Delta T} = 0.031\text{–}7.3;$$

$$P/P_c = 0.0045\text{–}0.98; \quad g/g_0 = 1\text{–}21.7.$$

It should be noted, however, that the agreement between the experimental data and the predicted results at near-critical pressures is rather surprising considering that a very simple method was used to allow for variation of the thermal properties of vapour, viz. these were related to the average film temperature. As is to be expected, the scatter of experimental data in the coordinates of Figs. 6 and 7 somewhat increases when $P/P_c > 0.8$, particularly at low temperature differences. Since to date only one paper [27] has furnished the data for pressures $P/P_c > 0.8$, additional experiments are obviously needed that the validity of the use of equations (47) and (49) in the near-critical region could be justified.

It is worth noting that the data presented in Fig. 5 can be successfully correlated by means of equation (47) alone, but at a sacrifice in the accuracy of calculations. In Table 2 are entered the comparative characteristics of various correlations derived on the basis of a complete set of experimental data [7, 13, 16, 18–21, 25, 27–33]. It is evident that the best result is obtained when both equations, (47) and (49), are used.

Equations (47)–(50), suggested in the present work, allow the prediction of heat transfer in the region where the intensity of the process does not depend on the size of the heating surface. The effect of the growth of the heat transfer rate in small-sized regions can be described, at least qualitatively, on the basis of the model of regular distribution of the sites of generation of bubbles.

To show this, let us consider a model of film boiling on a horizontal surface which is shown in Fig. 1. The

Table 2. Comparison of various correlations with experimental data

	Equations (47) and (49)	Equation (47) alone	Berenson's equation (2)	Frederking's equation (4)
The number of points beyond ± 25% interval (% to the total number)	6.5	8.5	21.6	22.3
Maximum deviation, %	+34 −35	+56 −34	+120 −28	+37 −47

magnitude of the heat flux, at least with a laminar flow of vapour film, can be determined as

$$q \sim \frac{\lambda}{\delta} \Delta T \frac{A_f}{A}, \tag{51}$$

where A_f/A is the relative area occupied by the vapour film on the heater. Then

$$\frac{A_f}{A} = 1 - \frac{A_b}{A}, \tag{52}$$

where A_b/A is a fraction of the surface occupied by vapour bubbles.

For a large enough surface with regular distribution of nucleation sites we can write

$$\frac{A_f}{A} = 1 - \frac{\frac{\pi}{4}(d_0)^2}{l_{D_\infty}^2/2}. \tag{53}$$

Taking into account that the departure diameter d_0 constitutes a certain fraction of the most dangerous wavelength, i.e.

$$d_0 = n l_{D_\infty}, \tag{54}$$

we shall obtain

$$\frac{A_f}{A} = 1 - \frac{\pi}{2} n^2. \tag{55}$$

Figure 8 shows A_f/A as a function of the number n. It should be noted that at $n = 0.565$ (which fairly well agrees with the experimental data of Hosler and Westwater [16]), A_f/A is equal to 0.499.

When the size of a flat horizontal surface is de-

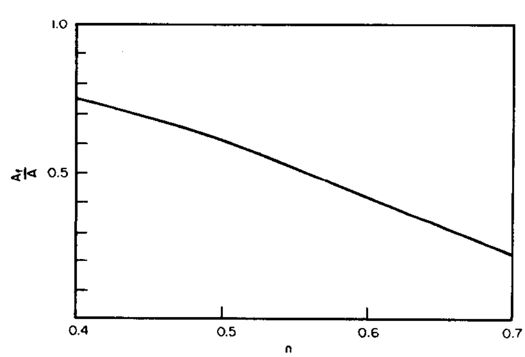

FIG. 8. Fraction of the heating surface occupied by a vapour film.

creased below a certain critical value [determined, according to [14], by equation (46)], the ratio A_f/A increases and, in conformity with (51), an increase is observed in the heat-transfer rate due to collapse of peripheral vapour bubbles on coming into contact with the surrounding walls or stagnant liquid layers. Finally, for the surface of the minimum size D, equal to $l_{D_\infty}/2$ or below, the case of $(A_f/A) \to 1$ is possible (which corresponds to a physical situation when the bubbles nucleate and collapse on the periphery of the heating surface). Thus, the maximum growth of the heat transfer rate on the surface of size $l_{D_\infty}/2$, in comparison with the 'large' surface, can amount to

$$\frac{(A_f/A)_{D=l_D/\sqrt{2}}}{(A_f/A)_{D\to\infty}} = \frac{1}{0.499} = 2.01. \tag{56}$$

It is of interest to compare this, rather approximate, analysis with experimental results. The experimental data reported by several authors are plotted in Fig. 9 as Nu/Nu_∞ vs D/l_{cr} [here Nu_∞ stands for the values calculated from formulae (47), (49)]. Firstly, there is a distinct effect of the size of a flat horizontal surface at $D/l_{cr} < 4$–5, which is in full conformity with equation (46). Secondly, if on log–log plot we join by a straight line the point $(2\sqrt{6}; 1)$ (condition of a 'large surface') to the point $(\sqrt{3}, 2.01)$, corresponding to expression (56), we shall obtain the relationship

$$\frac{Nu}{Nu_\infty} = 2.90\left(\frac{l_{cr}}{D}\right)^{0.67}, \tag{57}$$

which gives a satisfactory quantitative and qualitative agreement with experiment. A slightly larger scatter in the region of small sizes is explained, in our opinion, by the neglect of the dependence of the critical (or 'most dangerous') wavelength on the temperature difference. However, as this dependence is difficult to obtain (see above), it is hardly probable at the moment to refine (57) by taking into account the factor mentioned.

CONCLUSIONS

1. On the basis of the Reynolds analogy correlations (47)–(50) have been obtained predicting heat transfer in film boiling on a horizontal plate for a laminar and turbulent vapour flow in the film, with and without regard for friction on the liquid–vapour interface.

2. An investigation of the effect of vapour film thickness and vapour velocity in it on the 'most dangerous' wavelength of the Taylor instability under

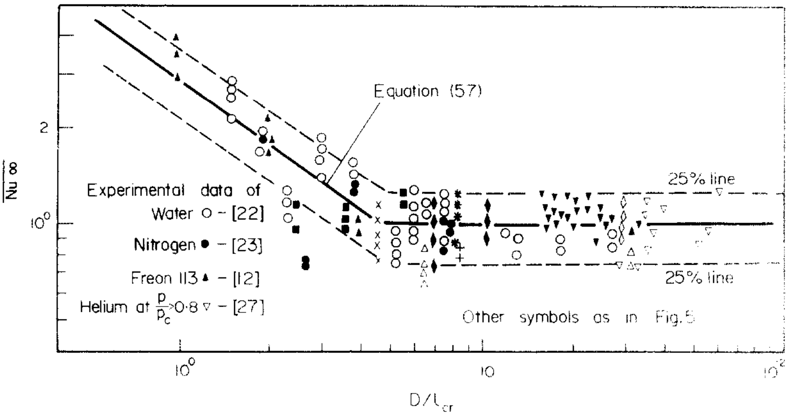

FIG. 9. Dependence of the heat-transfer rate on the size of a flat horizontal surface.

the conditions of film boiling has been made. It has been shown that in actual systems this influence is not very marked and, more often than not, it can be neglected in calculations.

3. Correlations (47)–(50) are shown to be the best ones to compare the results of the experiments on film boiling of nine various liquids with an accuracy of ±25%.

4. It has been shown that starting with a certain limit defined by expression (46), the heat transfer rate increases as the minimum size of the heating surface decreases. This effect is described by equation (57), obtained on the basis of the analysis of a change in the fraction of the heating surface occupied by a vapour film.

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EBULLITION EN FILM SUR UNE PLAQUE HORIZONTALE—UNE NOUVELLE FORMULATION

Résumé—On suggère une nouvelle approche de l'ébullition en film sur une surface horizontale dans un réservoir, à partir de l'analogie de Reynolds. Dans le cadre du modèle considéré, on étudie l'influence de l'épaisseur du film de vapeur et de la vitesse de la vapeur sur l'instabilité de Taylor pour l'interface. Quatre solutions limites ont été obtenues pour l'écoulement laminaire et turbulent de la vapeur dans le film avec ou sans prise en compte du frottement à l'interface liquide–vapeur. Les relations proposées unifient pratiquement tous les résultats expérimentaux disponislbes avec une marge de ± 25%. La frontière de la région où le transfert thermique dépend de la dimension de la surface chaude a été établie et une formule empirique rend compte de cet effet.

EINE NEUE GLEICHUNG FÜR DAS FILMSIEDEN AN EINER WAAGERECHTEN PLATTE

Zusammenfassung—Aufbauend auf der Reynolds-Analogie wird eine Gleichung für das Filmsieden an einer waagerechten Platte vorgeschlagen. Es wird der Einfluß der Dicke des Dampffilms und der Dampfgeschwindigkeit auf die Taylor-Instabilität der Grenzfläche untersucht. Es ergaben sich vier eingrenzende Lösungen: für laminare und turbulente Strömung im Film mit und ohne Reibung der Phasengrenzfläche zwischen Flüssigkeit und Dampf. Die vorgeschlagenen Gleichungen geben die vorhandenen Meßpunkte mit einer Unsicherheit von ± 25% wieder. Die Grenze des Bereichs, in dem der Wärmeübergang von der Größe der Heizfläche abhängt, wurde ermittelt und dafür eine empirische Gleichung angegeben.

ПЛЕНОЧНОЕ КИПЕНИЕ НА ГОРИЗОНТАЛЬНОЙ ПЛАСТИНЕ — НОВОЕ СООТНОШЕНИЕ

Аннотация — Предлагается подход к пленочному кипению на горизонтальной поверхности на основе аналогии Рейнольдса для случая обтекания пластины в продольном направлении. В рамках рассмотренной модели анализируется влияние толщины паровой пленки и скорости пара в ней на тейлоровскую неустойчивость межфазной поверхности раздела. Получены четыре предельных случая решения для ламинарного и турбулентного течения пара в пленке с учетом и без учета трения на границе раздела жидкость–пар. Предложенные соотношения обобщают практически весь имеющийся экспериментальный материал с точностью ±25%. Установлена граница зоны зависимости интенсивности теплообмена от размера поверхности нагрева, получена эмпирическая формула для учета этого эффекта.